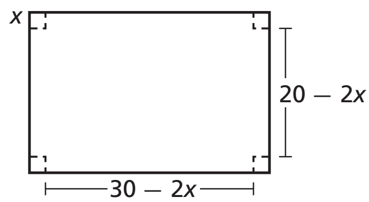
Name ______________________

Step It Out

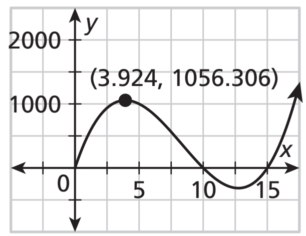
Learn the Math

EXAMPLE 1 Ariel is making a box out of sheet metal to contain her rock collection. The sheet of metal is 20 inches long and 30 inches wide. She cuts squares out of each corner and folds up the sides to build the box. What dimensions, to the nearest inch, maximize the volume of the box?



Write the equation for the volume of the box.

Squares of side length x inches are cut from each corner. So the height of the box after it is folded is x inches. Once the sides are folded up, the length of the box is given by $\ell = 20 - 2x$ and the width is given by $w = 30 - 2x$.



The volume is the product of the length, the width, and the height: $V = (20 - 2x)(30 - 2x)x$.

The value of x must be between 0 and 10 to ensure that none of the dimensions are negative.

Use a graphing calculator to find the height. Use the value of x to find the length and the width of the box. Then determine the maximum volume.

From the graph, $x \approx 4$ inches. So the length is approximately $20 - 2(4)$, or 12 inches, and the width is approximately $30 - 2(4)$, or 22 inches.

The maximum volume is $(12)(22)(4)$, or 1056 cubic inches.

Do the Math

Caleb is making a box out of cardboard to hold his tools. The piece of cardboard is 12 inches long and 15 inches wide. He cuts squares out of each corner and folds up the sides to build the box. What dimensions, to the nearest inch, maximize the volume of the box?

Write the equation for the volume of the box. Let ☐ represent the height of the box.

The length of the box is given by $l =$ ☐ $-$ ☐, and the width is given by $w =$ ☐ $-$ ☐. The volume is given by $V = ($☐ $-$ ☐$)($☐ $-$ ☐$)($☐$)$.

Use a graphing calculator to graph the volume function. Set the values of x between ☐ and ☐ to ensure that none of the dimensions are negative.

From the graph, $x \approx$ ☐ inches. The length is approximately ☐ inches, and the width is approximately ☐ inches. The maximum volume is $($☐$)($☐$)($☐$)$, or ☐ cubic inches.

Learn the Math

EXAMPLE 2 Dimitri bought a cylindrical clay flowerpot that holds approximately 15 gallons of dry soil $\left(1 \text{ gallon} \approx 0.15 \text{ cubic foot}\right)$. The radius of the pot is 9 inches. What is the height, to the nearest tenth of a foot, of the flowerpot?

Convert the radius to feet and the volume to cubic feet. Then use the formula for the volume of a cylinder: $V = \pi r^2 h$.

The radius is $\frac{9}{12}$, or 0.75 foot. Solve the proportion $\frac{1 \text{ gal}}{0.15 \text{ ft}^3} = \frac{15 \text{ gal}}{V}$ to convert the volume to cubic feet: $V = 15(0.15) = 2.25 \text{ ft}^3$.

$$V = \pi r^2 h$$

$$2.25 = \pi(0.75)^2 h$$

$$2.25 = 0.5625\pi h$$

$$\frac{2.25}{0.5625\pi} = h$$

$$h \approx 1.3 \text{ ft}$$

The height of the flowerpot is approximately 1.3 feet.

Do the Math

The diameter of a one-gallon cylindrical paint can is approximately 16.5 centimeters. Manufacturing specifications require there to be 1.27 centimeters of empty space above the level of the paint to allow for tinting and mixing. What height, to the nearest hundredth of a centimeter, must the paint can be?

$\left(1 \text{ gallon} \approx 3785.4 \text{ cubic centimeters}\right)$

Calculate the radius of the can, and substitute known values into the formula for the volume of a cylinder.

The radius is $\frac{\square}{\square}$, or $\square$ centimeters.

$$V = \pi r^2 h$$

$$\square = \pi\left(\square\right)^2 h$$

$$\square = \square\pi h$$

$$\frac{\square}{\square\pi} = h$$

$$h \approx \square \text{ ft}$$

To account for the extra space above the paint, add $\square$ centimeters to the height. The height of the can must be $\square$ centimeters.

Name ______________________

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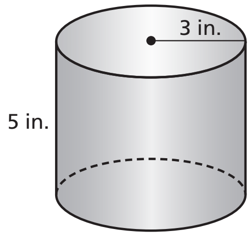
Find the volume of each figure. Round your answer to the nearest hundredth.

1.



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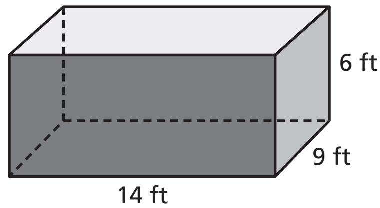
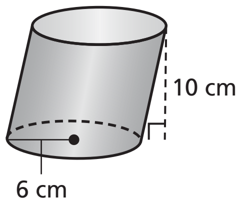


3. Math on the Spot Give exact answers in terms of π.

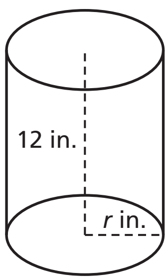
A. Find the volume of the cylinder.



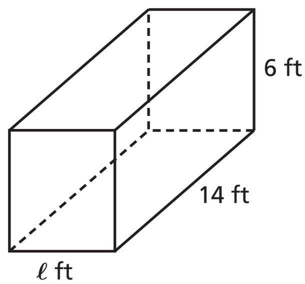
B. Find the volume of a cylinder with a base area of $25\pi\ \text{in}^2$ and a height equal to the radius.

Find the missing dimension of each figure. Round your answer to the nearest tenth.

4. $V = 100\ \text{in}^3$

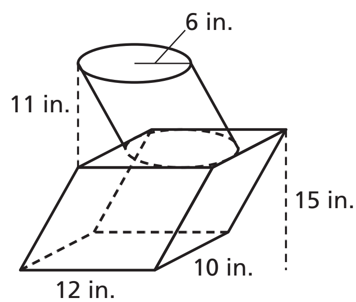


5. $V = 252\ \text{ft}^3$

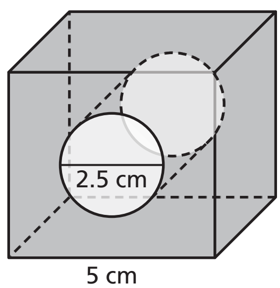


Find the volume of each composite figure. Round your answer to the nearest tenth.

6.

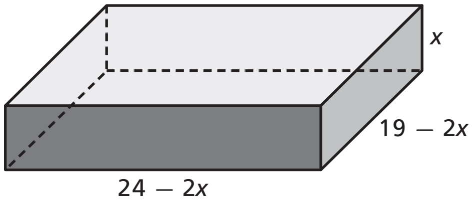


7. A cylindrical-shaped hole is cut from the center of a cube.



Find the maximum volume of each box and the dimensions, in inches, that produce the maximum volume. Round all answers to the nearest hundredth.

8.



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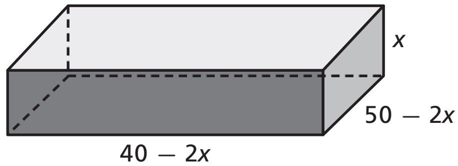


10. Critique Reasoning Eugene believes that a prism with a square base of 14 centimeters and a height of 9 centimeters will have the same volume as a cylinder with a diameter of 14 centimeters and a height of 9 centimeters. Is Eugene correct? Explain your reasoning.

11. Reason A rectangular in-ground pool with a uniform depth measures 36 feet long by 18 feet wide. The fill line is 1.5 feet below the pool deck. The pool is filled with approximately 24,236.88 gallons of water $(1 \text{ gallon} \approx 0.1337 \text{ cubic feet})$. How far underground is the bottom of the pool?

12. Jerra wants to store 16 ounces of coffee beans in a cylindrical canister. The radius of the canister is 7 centimeters. $(1 \text{ ounce} \approx 29.57 \text{ cubic centimeters})$ What is the least height, to the nearest tenth of a centimeter, the canister can be to hold all of the coffee beans?

13. Open Ended Give the dimensions of a rectangular prism and of a cylinder so that the two figures have approximately the same volume.

14. The quarter is a circular coin with a diameter of 24.26 millimeters and a thickness of 1.75 millimeters. What is the volume of a stack of 20 quarters to the nearest cubic millimeter?

Ⓐ 8090

Ⓑ 16,179

Ⓒ 32,357

Ⓓ 64,714

Name

Step It Out

Learn the Math

EXAMPLE 1 A ski resort creates a three-dimensional model of its logo consisting of two cones. One of the cones has a radius of 6 feet and a height of 12 feet, and the other has a radius of 5 feet and is $\frac{2}{3}$ the height of the first. What is the volume of the model? Round to the nearest hundredth.

Find the height of the second cone: $12 \cdot \frac{2}{3} = 8$ feet.

Use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of each cone. Substitute the values for the radius and the height of each cone into the formula. Remember that volume is measured in cubic units.

First Cone:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(6)^2(12)$

$V = 144\pi$

$V \approx 452.39 \text{ ft}^3$

Second Cone:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(5)^2(8)$

$V = \frac{200\pi}{3}$

$V \approx 209.44 \text{ ft}^3$

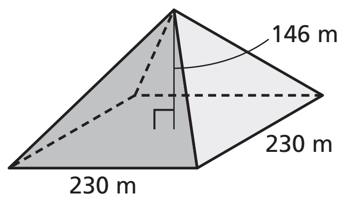
Add the volumes of the two cones together to find the volume of the model.

$144\pi + \frac{200\pi}{3}$

The total volume of the model is approximately 661.83 ft^3.

Do the Math

An Egyptian pyramid with a square base with a side length of 230 meters once stood at 146 meters tall. Due to erosion, it is now only 139 meters tall but the base is unchanged. How much volume did the pyramid lose due to erosion? Round to the nearest whole.



Calculate the area of the base B.

$230 \cdot 230 = \square$

Use the equation for volume of a pyramid, $V = \frac{1}{3}Bh$.

Find the volume of the pyramid before and after erosion.

Volume Before:

$V = \frac{1}{3}(\square)(146)$

$V \approx \square$

Volume After:

$V = \frac{1}{3}(\square)(\square)$

$V \approx \square$

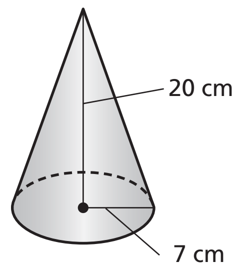
Subtract the volumes to find how much was lost.

$\square - \square = \square$ m^3 was lost due to erosion.

Learn the Math

EXAMPLE 2 A plastic cone has a radius of 7 centimeters and a height of 20 centimeters. The cone has a mass of 900 grams. What is the density of the plastic? Round to the nearest hundredth.

Find the volume of the cone using the formula $V = \frac{1}{3}\pi r^2 h$.



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (7)^2(20)$

$V \approx 1026.25 \text{ cm}^3$

Find the density using the formula density $= \frac{\text{mass}}{\text{volume}}$.

$d = \frac{900}{1026.25} \approx 0.88$

The density of the plastic is about 0.88 gram per cubic centimeter.

Do the Math

The head of a steel meat tenderizer is a cube with a side length of 5 centimeters. One face of the cube contains 25 square pyramids that protrude from the surface. The side length of the base of each square pyramid is 0.5 centimeter, and the height of each is 0.5 centimeter. The mass of the head of the tenderizer is 1000 grams. What is the density of the steel? Round to the nearest hundredth.

Find the volume of the cube using the formula $V = s^3$.

$V = \boxed{}^3 = \boxed{}$

Find the area of the base B of each pyramid.

$B = \left(\boxed{}\right)^2 = \boxed{}$

Find the volume of one pyramid using the formula $V = \frac{1}{3}Bh$.

$V = \frac{1}{3}\left(\boxed{}\right)\left(\boxed{}\right) \approx \boxed{}$

Multiply the volume of one pyramid by the number of pyramids. Then add the volume of the cube to find the total volume.

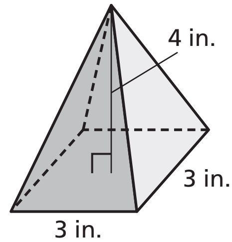
$25\left(\boxed{}\right) + \boxed{} \approx \boxed{}$

Find the density using the formula density $= \frac{\text{mass}}{\text{volume}}$.

$d = \frac{1000}{\boxed{}} \approx \boxed{}$

The density of the steel is about $\boxed{}$ grams per cubic centimeter.

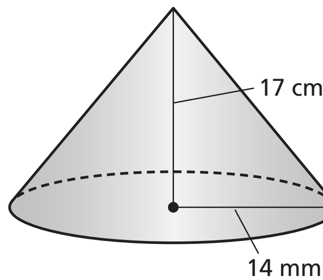
1. Find the volume of the pyramid.



2. Find the volume of a cone with a height of 3 feet and a radius of 1 foot. Round to the nearest hundredth.

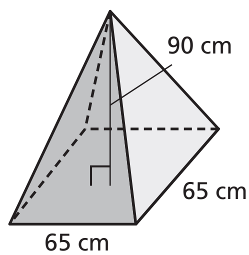
3. Find the volume of a triangular pyramid with a height of 10 centimeters. The base of the pyramid has a height of 14 centimeters and a base length of 12 centimeters.

4. Find the volume of the cone shown. Round to the nearest hundredth.



5. Find the density of a metal cone with a height of 20 centimeters and a radius of 5 centimeters. The mass is 2500 grams. Round to the nearest hundredth.

6. Find the density of the wooden square pyramid shown. The mass is 100,000 grams. Round to the nearest hundredth.



7. Find the density of a plastic triangular pyramid with a height of 5 feet. The base of the pyramid has a height of 4 feet and a base length of 3 feet. The mass is 55 kilograms.

8. Find the density of a wooden cone with a height of 100 centimeters and a radius of 25 centimeters. The mass is 18,000 grams. Round to the nearest hundredth.

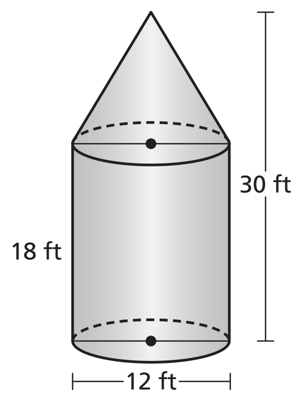
9. **Math on the Spot** Find the volume of the composite figure. Round to the nearest hundredth.

A. Find the volume of the cylinder.

B. Find the height of the cone.

C. Find the volume of the cone.

D. Find the total volume.



10. Reason A rubber ball consists of a sphere with 150 small cones protruding from the surface. The volume of the sphere is 50 cubic centimeters. Each cone has a radius of 0.2 centimeter and a height of 0.4 centimeter.

A. Write the formula for the total volume of the ball.

____

B. What is the volume of the ball? Round to the nearest hundredth.

____

11. A square pyramid made of metal is at the peak of the Washington Monument in Washington, DC. It was constructed of the metal considered to be most precious at the time. The side length of the base of the pyramid is 11.37 centimeters, and the height is 22.86 centimeters. The mass of the pyramid is about 2835 grams. The following are approximate densities of various metals in grams per cubic centimeter: lead 11.36; aluminum 2.70; silver 10.49; gold 19.32; zinc 7.13.

A. What is the volume of the pyramid?

____

B. What is the density of the pyramid?

____

C. Which type of metal is the pyramid made of?

____

12. The area of the base of a hexagonal pyramid is 200 square inches. The volume of the pyramid is double the volume of a cone with a radius of 6 inches and a height of 8 inches. What is the height of the pyramid to the nearest inch?

Ⓐ 3 inches

Ⓑ 5 inches

Ⓒ 9 inches

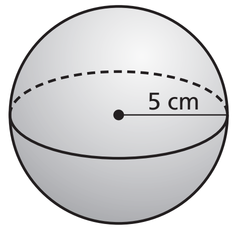
Ⓓ 27 inches

Name ____________________

Step It Out

Learn the Math

EXAMPLE 1 When a spaceship is orbiting Earth, water can be suspended in the air and it will form a sphere. Amaya measures a water sphere on her spaceship that has a radius of 5 centimeters. Find the volume and the mass of the water. To find the mass, use the fact that the density of water is 1 gram per cubic centimeter. Round answers to the nearest tenth.



Use the formula for volume of a sphere, $V = \frac{4}{3}\pi r^3$.

$$\begin{aligned} V &= \frac{4}{3}\pi(5)^3 \\ &= \frac{500}{3}\pi \\ &\approx 523.6 \text{ cm}^3 \end{aligned}$$

Use the formula for density, $d = \frac{m}{V}$, to calculate the mass.

$$\begin{aligned} m &= dV \\ &\approx (1)(523.6) \\ &\approx 523.6 \text{ g} \end{aligned}$$

Do the Math

When a substance has a density that is less than the density of water, it floats. A wooden sphere has a radius of 4 centimeters and a mass of 250 grams. Will the wooden sphere float? Round answers to the nearest hundredth.

Calculate the volume of the wooden sphere.

$$V = \frac{\square}{\square}\pi r^3 = \frac{\square}{\square}\pi \square^3 = \frac{\square}{3}\pi$$

$$\approx \square \text{ cm}^3$$

Calculate the density of the wooden sphere.

$$d = \frac{\square}{V} \approx \frac{250}{\square}$$

$$\approx \square \text{ g/cm}^3$$

Because the density of the wooden sphere is ____________ than the density of water, it ____________.

Learn the Math

EXAMPLE 2 Planets are nearly spherical. They are not perfectly spherical because their equatorial diameters are greater than their polar diameters. The equatorial diameter of Earth is 7926 miles. The equatorial diameter of Mars is 4222 miles. Estimate how many times as great the volume of Earth is as the volume of Mars.

Use the formula for the volume of a sphere to estimate the volumes of the two planets.

Earth

$$V = \frac{4}{3}\pi(3963)^3$$
$$\approx 2.61 \times 10^{11} \text{ mi}^3$$

Mars

$$V = \frac{4}{3}\pi(2111)^3$$
$$\approx 3.94 \times 10^{10} \text{ mi}^3$$

Divide the volumes to determine how many times as large Earth is as Mars.

$$\frac{2.61 \times 10^{11}}{3.94 \times 10^{10}} \approx 6.6$$

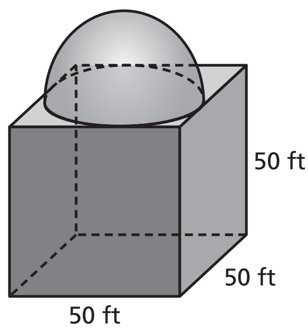
The volume of Earth is about 6.6 times as great as the volume of Mars.

Do the Math

Kelly has a Granny Smith apple with a diameter of 7.5 centimeters and a Honeycrisp apple with a diameter of 9 centimeters. The apples are nearly spherical. Estimate how many times as great the volume of the Honeycrisp apple is as the volume of the Granny Smith apple. Round to the nearest tenth.

Learn the Math

EXAMPLE 3 A storage shed is composed of a cube with a hemisphere on top, as shown. The diameter of the hemisphere is equal to the side length of the cube. What is the volume of the shed to the nearest cubic foot?



Use the formula $V = s^3$ to calculate the volume of the cube.

$$V = 50^3 = 125{,}000 \text{ ft}^3$$

A hemisphere is half a sphere, so use the formula $V = \frac{2}{3}\pi r^3$ to calculate the volume of the hemisphere.

$$V = \frac{2}{3}\pi(25)^3$$
$$\approx 32{,}725 \text{ ft}^3$$

The volume of the shed is about $125{,}000 + 32{,}725$, or $157{,}725$ ft^3.

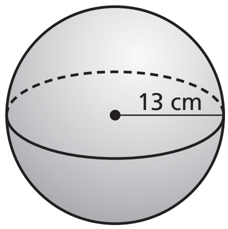
Do the Math

A gas tank is composed of a cylinder with a hemisphere on each end. The height of the cylinder is 10 feet, and the radius is 2 feet. The bases of the hemispheres are congruent to the base of the cylinder. What is the volume of the gas tank to the nearest cubic foot?

Name ______________________________

Find the volume of each figure. Round your answer to the nearest tenth.

1.



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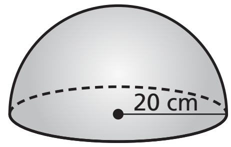


3. a sphere with a radius of 7 feet

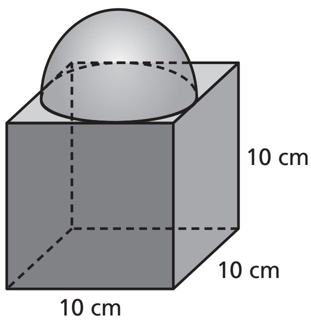
4. a sphere with a diameter of 18 inches

5. a sphere with a diameter of 12 meters

6. a hemisphere with a diameter of 8 feet

Find the volume of each composite figure. Round your answer to the nearest tenth.

7.



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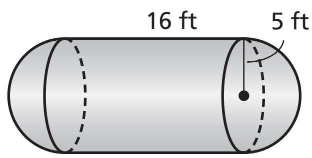

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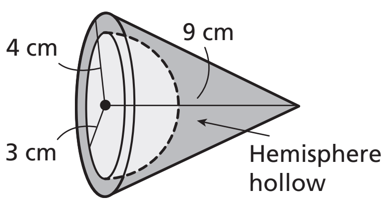

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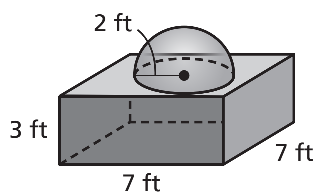
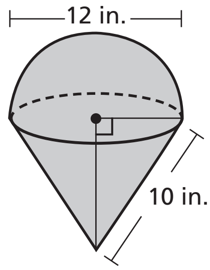


11. Math on the Spot Find the volume of the composite figure. Round to the nearest cubic inch.



12. Reason The diameter of Earth's moon is about 2159 miles. Like Earth, it is nearly spherical. Use the information given in Example 2 to estimate how many moons could fit inside Earth.

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13. Tyler is designing a bouncy object for his toy manufacturing company. The object consists of a cube with hemispheres protruding from each face. The side length of the cube and the diameters of the hemispheres all measure 6 inches. What is the volume of the object to the nearest cubic inch?

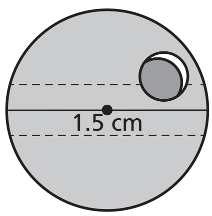
---

14. A cylindrical container holds three tennis balls, each with a radius of 3.3 centimeters. The radius of the container is 3.5 centimeters, and the length is 20.2 centimeters. What is the volume of the empty space in the container to the nearest tenth?

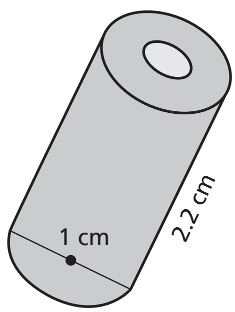
---

15. Critique Reasoning Students are making jewelry in art class. Andi is drilling through the center of holes wooden spheres to make beads for a necklace. (Note: because the sphere is round, the holes drilled through it are not quite cylinders because the "bases" are not flat.) Clara is drilling cylindrical holes into wooden cylinders to make beads for a bracelet. Clara says the volumes of their beads are the same, but Andi says the volume of her beads is greater than the volume of Clara's beads. The holes have radius 0.25 centimeter. Who is correct? Explain.

Andi's bead



Clara's bead



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16. Patrick has to calculate the volume of 16 hemispheres with the same radius. To simplify the calculations, he computes the volume of one sphere with the given radius. By what number does Patrick need to multiply the volume of the sphere to determine the total volume?

Ⓐ 2

Ⓑ 4

Ⓒ 8

Ⓓ 16