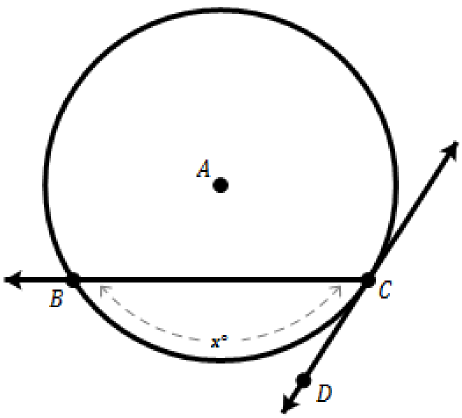
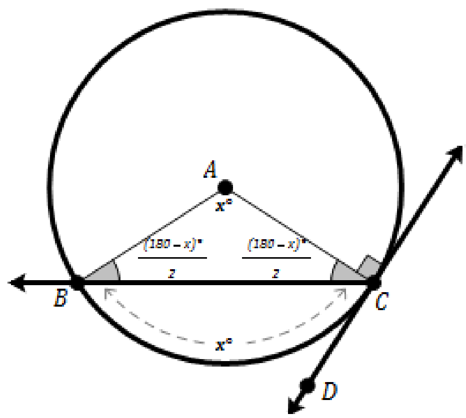
Sec 4.3 – Angles of Circles Name:

Tangent Line Angles



Consider the tangent line $\overleftrightarrow{DC}$ and the ray $\overrightarrow{CB}$ which intercepts arc $\widehat{BC}$ and has a measure of x°.

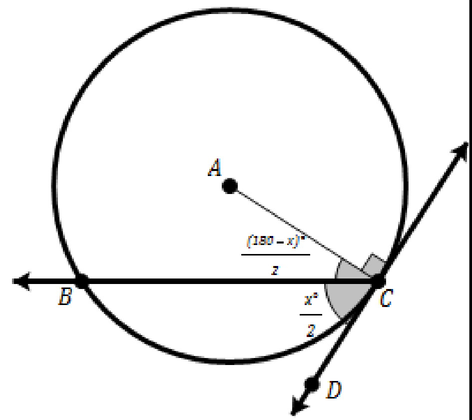


Draw an auxiliary segment $\overline{AB}$ and $\overline{AC}$ to create an isosceles triangles. We know that $m\measuredangle A = x°$ as a central angle and the interior angles of ΔABC sum to 180˚.

So, $m\measuredangle B + m\measuredangle C = 180° - x$

Also, $m\measuredangle B = m\measuredangle C$ because they are the base angles of an isosceles triangle. So, $m\measuredangle B + m\measuredangle B = 180° - x$ which simplifies: $2 \cdot m\measuredangle B = 180° - x$ or

$$m\measuredangle B = \frac{180°-x}{2}$$



Finally, $\overline{AC}$ must be perpendicular to $\overleftrightarrow{DC}$ since it is tangent of the circle. The angle $\measuredangle DCB$ & $\measuredangle ACB$ must sum to 90˚. So, we can find

$$m\measuredangle DCB = 90 - \frac{180° - x}{2}$$

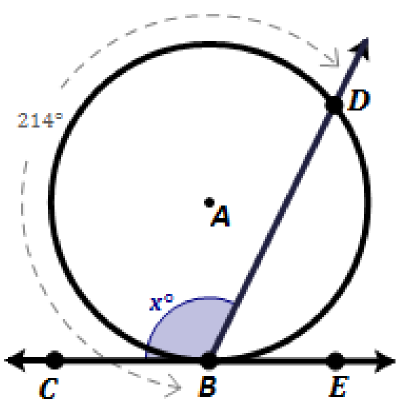
which simplifies:

$$m\measuredangle DCB = \frac{x}{2}$$

"The measure of an angle formed by a tangent and a chord drawn to the point of tangency is exactly ½ the measure of the intercepted arc."

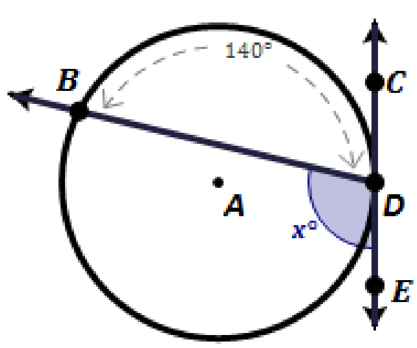
Find the most appropriate value for 'x' in each of the diagrams below. (Assume CE is tangent to the circle.)

1.



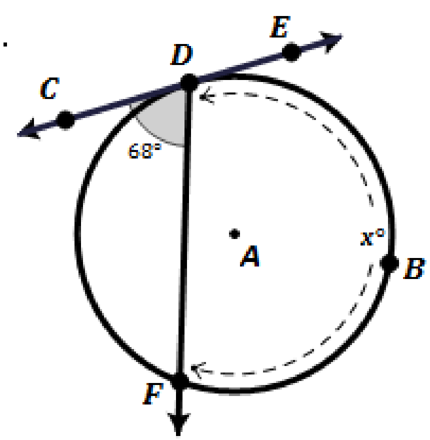
$x =$

2.



$x =$

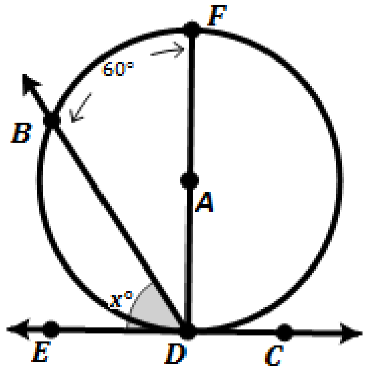
3.



$x =$

Find the most appropriate value for 'x' in each of the diagrams below. (Assume CE is tangent to the circle.)

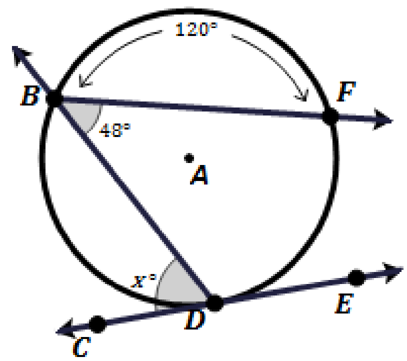
4.



(You may assume DF is a diameter.)

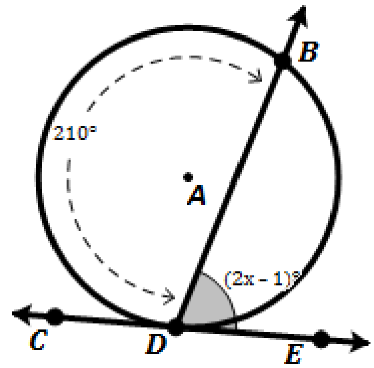
$x =$

5.



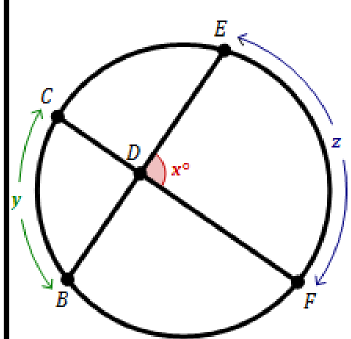
$x =$

6.

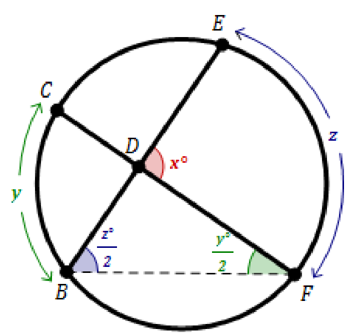


$x =$

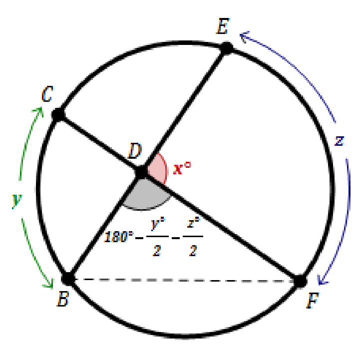
Intersecting Chords Interior Angles



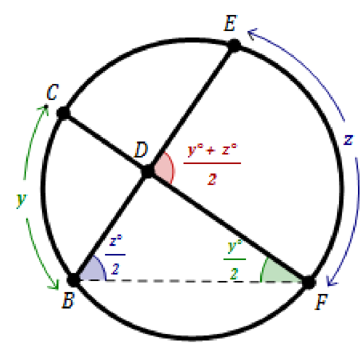
Consider the intersecting chords $\overline{BE}$ and $\overline{FC}$ that intercept the arcs $\widehat{CB}$ and $\widehat{FE}$.



Draw an auxiliary segment $\overline{BF}$ to create the inscribed angles that we know are half of the intercepted arc. So, $m\measuredangle FBE = \frac{z°}{2}$ and $m\measuredangle BFC = \frac{y°}{2}$



Since triangles interior angles sum to 180˚. So we can subtract the 2 angles of triangle ΔDBF to find angle $m\measuredangle BDF = 180° - \frac{y°}{2} - \frac{z°}{2}$

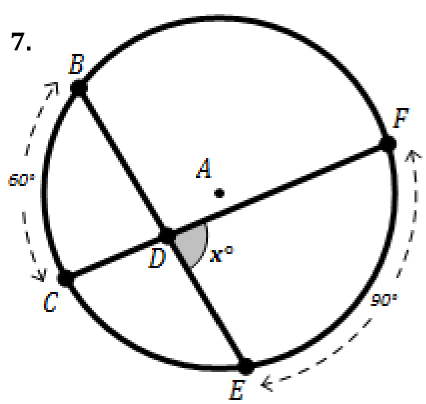


Finally, since $\measuredangle FDE$ forms a linear pair with $\measuredangle BDF$ we can subtract from 180˚ to find: $m\measuredangle FDE = 180° - \left(180° - \frac{y°}{2} - \frac{z°}{2}\right)$ which simplifies to $m\measuredangle FDE = \frac{y°+z°}{2}$

"The measure of an angle formed by two intersecting chords of the same circle is exactly ½ the measure of the sum of the two intercepted arcs."

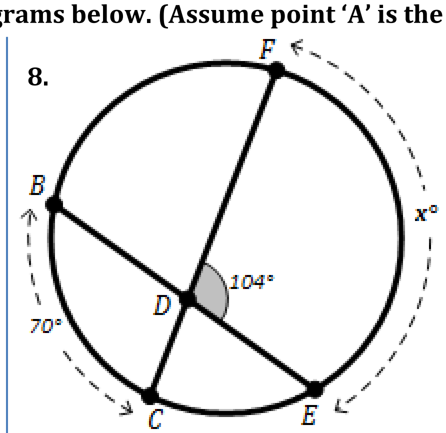
Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)

7.



$x =$

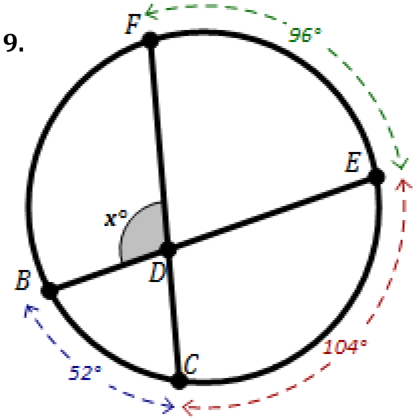
8.



$x =$

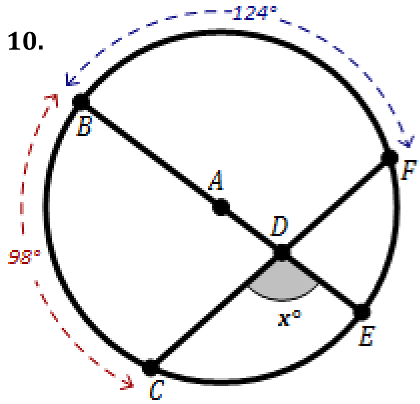
Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)

9.



$x =$

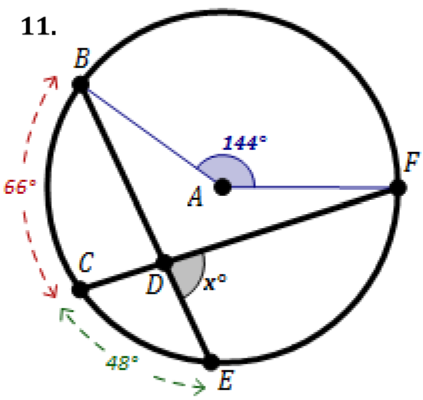
10.



(You may assume BE is a diameter.)

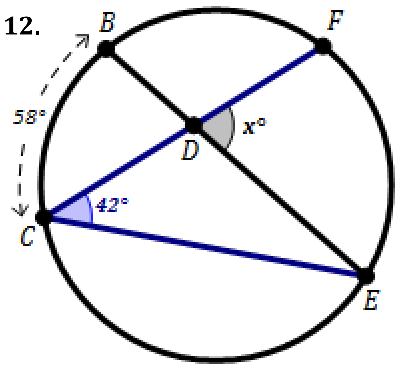
$x =$

11.



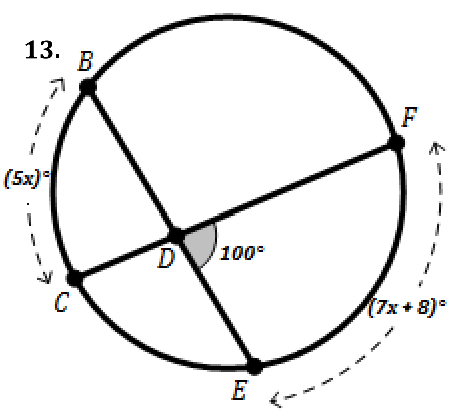
$x =$

12.



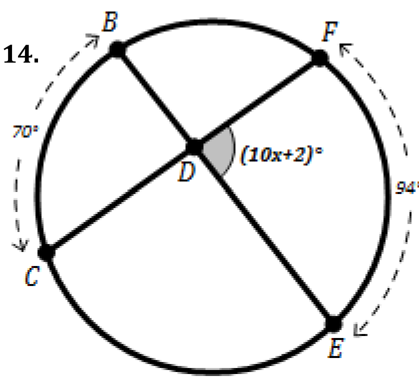
$x =$

13.



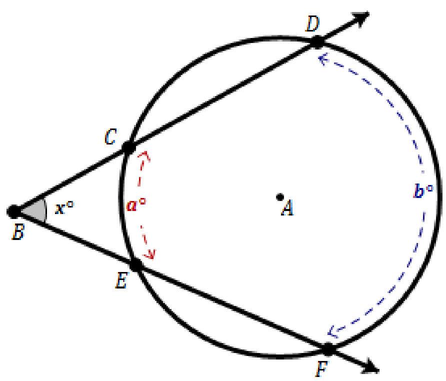
$x =$

14.

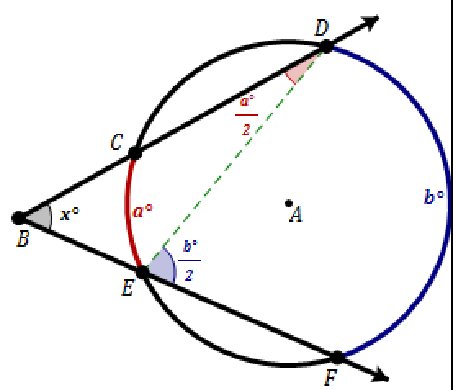


$x =$

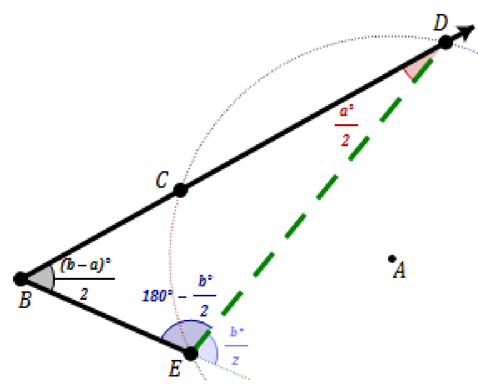
Secant Lines Exterior Angle



Consider the rays $\overrightarrow{BD}$ and $\overrightarrow{BF}$ which intercepts arc $\widehat{CE}$ and $\widehat{FD}$ which measure a° and b° respectively.



Draw an auxiliary segment $\overline{ED}$. We know that $m\measuredangle DEF = \frac{b^\circ}{2}$ and $m\measuredangle CDE = \frac{a^\circ}{2}$ because each is an inscribed angle.

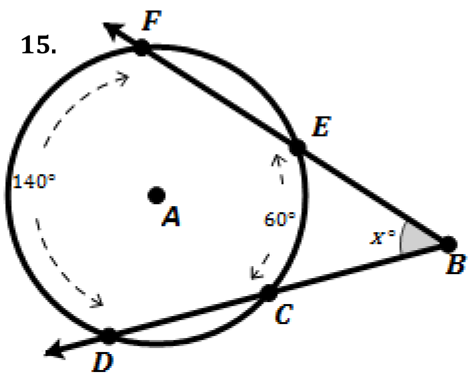


Finally, $m\measuredangle DEB = 180^\circ - \frac{b^\circ}{2}$ since the two angles at point E forma linear pair. Furthermore, $m\measuredangle B = 180^\circ - \left(180^\circ - \frac{b^\circ}{2}\right) - \frac{a^\circ}{2}$ since a triangle's interior angles sum to 180° and that would simplify to $m\measuredangle B = \frac{(b-a)^\circ}{2}$.

"The measure of an angle formed on the exterior of a circle by two intersecting secants of the same circle is exactly ½ the measure of the difference of the two intercepted arcs."

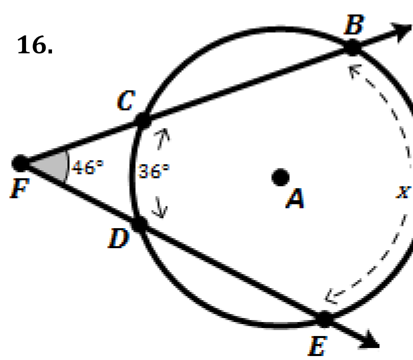
Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)

15.



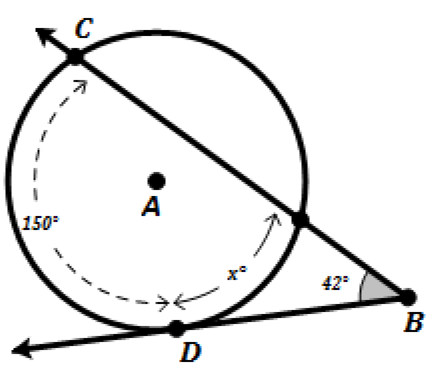
$x =$

16.



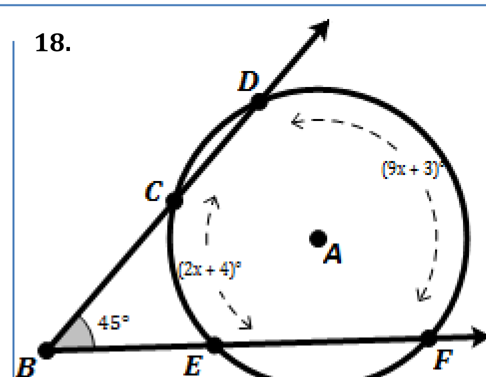
$x =$

17.



$x =$

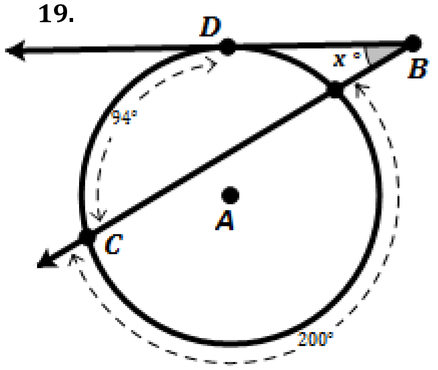
18.



$x =$

Find the most appropriate value for 'x' in each of the diagrams below. (Assume point 'A' is the center of the circle.)

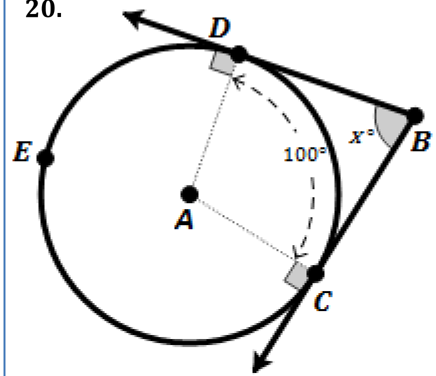
19.



You may assume DB is tangent to the circle.

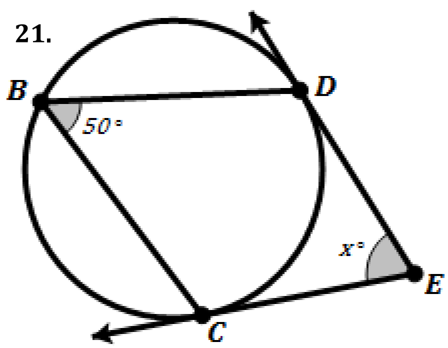
$x =$

20.



$x =$

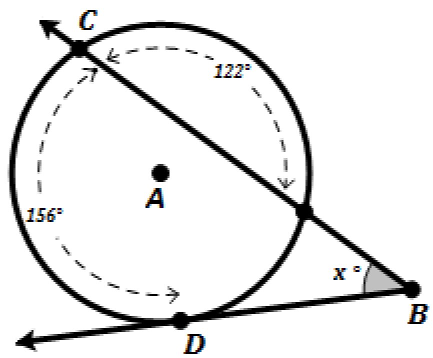
21.



You may assume EC and DE are tangent to the circle.

$x =$

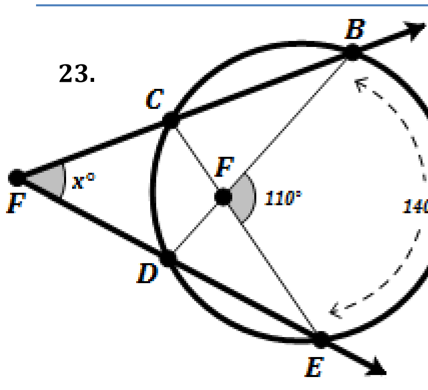
22.



You may assume DB is tangent to the circle.

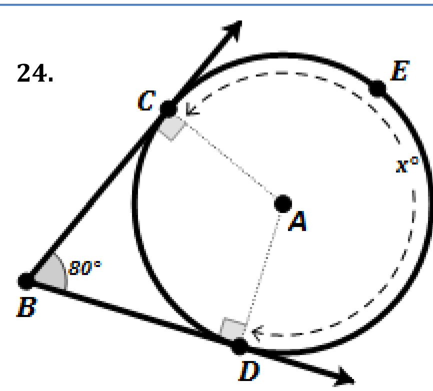
$x =$

23.



$x =$

24.



$x =$